Topology and the Baryon-Antibaryon Asymmetry in the Early Universe

Gerald E. Marsh
Argonne National Laboratory (Ret)
gemarsh@uchicago.edu

ABSTRACT

It is currently believed that baryon number conservation must be violated to produce a preponderance of baryons over anti-baryons in the early universe. An alternative to violating baryon number conservation is given here that preserves CPT invariance. There are two Appendices that cover some elements of The Big Bang Model and CPT Invariance.

Introduction

The baryon-antibaryon asymmetry in the early universe cannot be explained within the Standard Model of particle physics and cosmology. The idea of breaking baryon number symmetry between baryons and anti-baryons—resulting in *C* and *CP* invariance being violated—so as to obtain a preponderance of matter over antimatter is fraught with problems. Another approach is needed. One is offered below. It involves topological change, which has usually been ruled out, but the arguments against it have been found to be wanting. This is discussed in some detail below. We begin with the topology.

The FRW metric and 3-spheres

Seconds after the Big Bang (see Appendix 1), the universe can be accurately described by an approximately spatially-flat, radiation-dominated FRW metric. The designation FRW comes from Robinson and Walker finding that the Friedman-Lemaître metric can be put into the form

$$ds^2 = -dt^2 + S^2(t)d\sigma^2,$$

where, in spherical coordinates $d\sigma^2$ is given by

$$d\sigma^2 = \left(d\chi^2 + f^2(\chi)(d\theta^2 + \chi^2(d\theta^2 + sin^2\theta\ d\phi^2)\right).$$

Eq. (1)

This metric is a 3-dimensional hypersurface of constant curvature independent of time. Equation (1) can have three values for $f(\chi)$ depending on the normalized curvature *K* of the universe obtained by rescaling the function *S*:

$$f(\chi) = \left(\begin{array}{c} sin\chi\ if\ K = +1 \\ \chi\ if\ K = 0 \\ sinh\chi\ if\ K = -1 \end{array} \right. .$$

Eq. (2)

If K = 0 or −1, the 3-dimensional hypersurfaces are diffeomorphic to three-dimensional flat spaces and if K = +1 it is diffeomorphic to the three-sphere $\mathbb{S}^3$. For K = 0 or −1, $0 \leq \chi \leq \infty$, and for K = +1, $0 \leq \chi \leq 2\pi$.

Current models of the early universe include inflation, which smooths out any inhomogeneities and anisotropies thereby reducing the curvature of space. Current astronomical observations and measurements constrain the spatial curvature to be very close to zero, but cannot determine the sign of the curvature if it exists.

The case of a very slight positive curvature will be considered here, in which case Eq. (1) describes the spatial geometry of a 3-sphere $\mathbb{S}^3$. A 3-sphere is a compact manifold and while being finite does not have a boundary. This choice is made because it offers a possible explanation for the preponderance of baryons over anti-baryons in the early universe.

The 3-sphere can be seen to be the union of two 3-balls by a homeomorphism[1] h, which maps the boundary of B_1^3 onto the boundary of B_2^3. This is illustrated in Fig. 1.

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[1] R. H. Bing, *The Geometric Topology of 3-Manifolds* (The American Mathematical Society, Rhode Island 1983), Colloquium Publications, v. 40. Homeomorphic spaces are "the same" from a topological point of view and satisfy the conditions that it is a bijection (one-to-one and onto), it is continuous, and the inverse function is continuous; i.e., an open mapping.

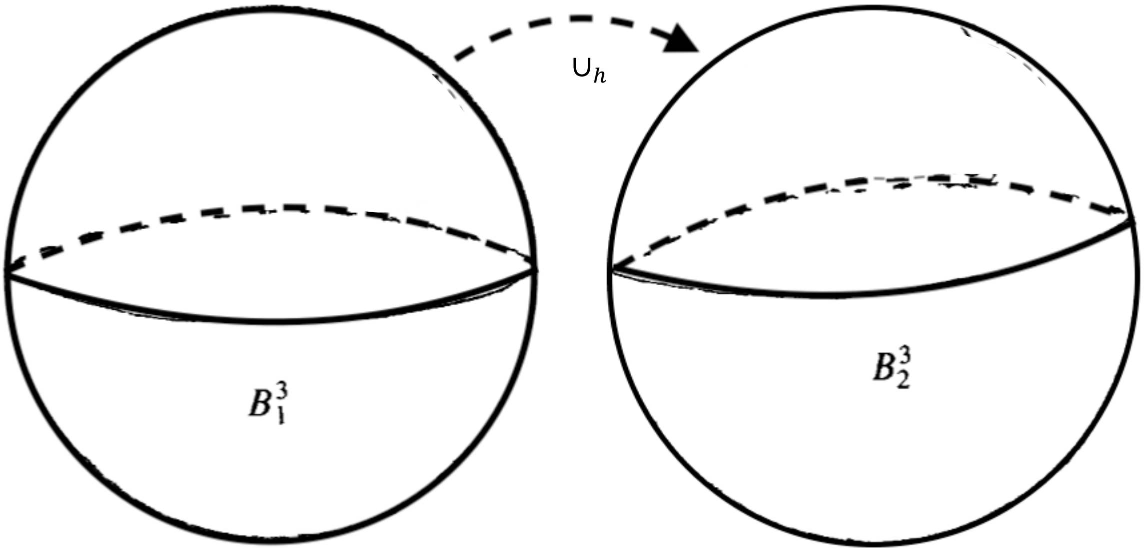

Figure 1. The 3-sphere $\mathbb{S}^3$ as a union of two 3-balls by a homeomorphism h. $B_1^3 and B_2^3$ designate the two 3-balls. If a point p is contained in the interior of B_i^3, where $i = 1$ or 2, then any open set containing p is a neighborhood of p in $B_1^3 \cup_h B_2^3$.

$\mathbb{S}^3$can also be thought of as the one-point compactification of 3-dimensional space $\mathbb{R}^3$, obtained by adding the point at infinity, but this definition will not be used here.

Separation of the Matter and Antimatter Universes from $\mathbb{S}^3$

What is proposed here is the separation of the early $\mathbb{S}^3$ universe into a matter and separate antimatter universe. This would occur occurs at a time ~4.7×10^4 yr after the big bang at the end of the radiation-dominated period and before quark confinement and baryon-antibaryon annihilation occurs in the conventional theory. The time evolution in the matter and antimatter universes, while being different, continue to follow the usual laws of physics. For example, particle and antiparticle pairs could still be created and the Feynman interpretation of an antiparticle as a particle moving backwards in time would still hold.

This proposed separation of the universe into a matter and antimatter universe at ~4.7 × 10^4 yr after the big bang preserves the *CPT* theorem (see Appendix 2) and avoids the problems associated with the idea of breaking the baryon number symmetry.

Boyle, Finn, and Turok[2] have also proposed a separation of the universe into a matter and antimatter universe. They propose that the universe immediately after the big bang became a universe/anti-universe pair that directly emerges into the radiation-dominated era. They reason that during the radiation-dominated era the metric is $g_{\mu\nu} \propto \tau^2 \eta_{\mu\nu}$ where $\eta_{\mu\nu}$ is the Minkowski metric and τ is the conformal time. They then note that the metric has time reversal symmetry under $\tau \to -\tau$. This is the basis for their proposal that the universe is a universe/anti-universe pair that directly emerges after the big bang. They also couple their theory with the Feynman Stückelberg interpretation of antiparticles being particles moving backwards in time.[3] This is very different from the proposal being made here that the division into a matter and antimatter universes is based on topological change at a time ~4.7 × 10^4 yr after the big bang at the end of the radiation-dominated period.

Figure 1 showed how the 3-sphere can be illustrated as the union of two 3-balls. Now consider the reverse process where the inverse of the homeomorphism *h* is applied to the 3-sphere. This is shown in Fig. 2.

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[2] L. Boyle, K. Finn, and N. Turok, "CPT-Symmetric Universe", *Phys. Rev. Lett.* **121** (2018), p. 251301. See also their longer paper available at arXiv:1803.08930.

[3] In the general view of time, moving backwards in time would take one to a three-dimensional space as it was in the past with the configuration of matter being what it was at each instant of time in the past. In this conception of time, three-dimensional hypersurfaces continue to exist in the sense that moving backward in time recapitulates three-dimensional space exactly as it was in the past. However, this usual conception of time imposes our own psychological time on the past, it does not mean that the physical past actually continues to exist. The Feynman-Stückelberg conception of antiparticles is fully consistent with the non-existence of past three-dimensional hypersurfaces.

Since Fig. 2 is meant to represent the early universe undergoing a topological change from a 3-sphere to two 3-balls some ~4.7 × 10^4 yr after the big bang at the end of the radiation-dominated period, the dimension of time, in the form of the function $S(t)$ from Eq. (1), must be introduced: the 3-sphere and both balls are then assumed to be expanding at the same rate. While the coordinate of time does not itself have an intrinsic orientation asymmetry, or arrow associated with it, the expansion of the universe does impose a direction in time. The 3-sphere, B_1^3, and B_2^3 have time moving in the same direction.

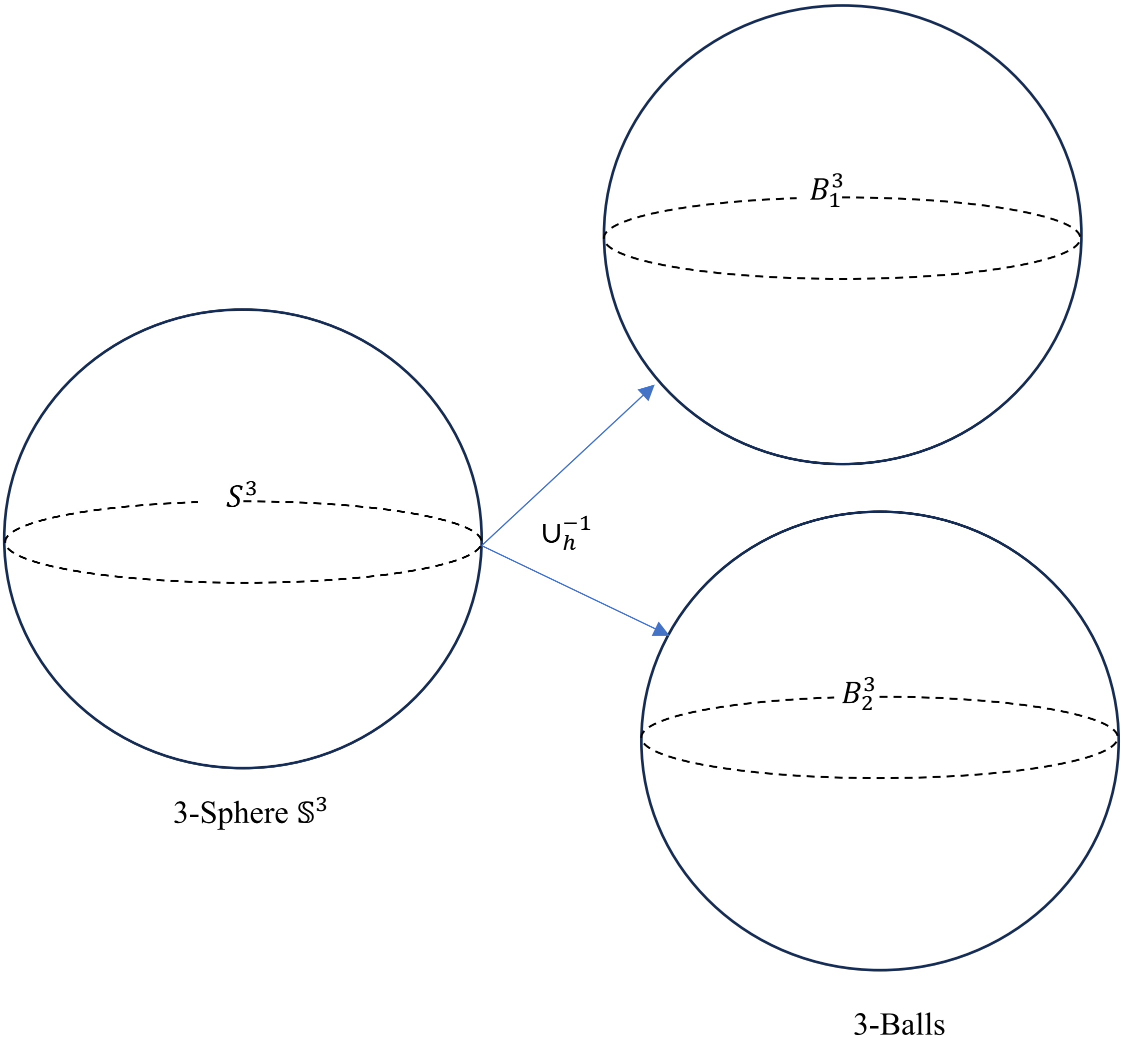

Figure 2. A 3-sphere $\mathbb{S}^3$ becomes two 3-balls under the inverse of the homeomorphism h used in Fig. 1. In general, a "ball" may be open or closed. An open ball of radius r is the set of all points at a distance strictly less than r from the ball's center.

Just after ~4.7×10^4 yr after the big bang, when the quark-gluon plasma comes into existence at the end of the radiation dominated period, is when the topological change is assumed to occur, there is a division at that time such that quarks are contained in say B_1^3 and antiquarks in B_2^3.

The key questions are: Is topological change consistent with general relativity; and, how does the separation of quarks and antiquarks occur?

With regard to topological change, Geroch[4] showed that for any spacetime containing *two separate* spacelike hypersurfaces of different topology there would have to be either closed timelike curves or singularities. In addition, Tipler[5] later showed that topology changing spacetimes are singular. In 1991, however, Horowitz[6] noted that the theorems of Geroch and Tipler basically showed that if a metric satisfies the Einstein field equation on a topologically changing manifold it cannot be well defined everywhere. He showed that the points at which the metric is ill defined do not have strong curvature singularities. To put it in his words, "The singularities can be very mild. In fact, they can be so mild that in some sense they are not there at all!" In particular, the curvature scalars are all finite and the curvature on the manifold does not diverge as one approaches the degenerate points.

Without invoking quantum gravity, Borde[7] found a way that classical general relativity can allow topological change and that is to eliminate the requirement that foliation of spacetime

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[4] R. Geroch, *J. Math. Phys*. **8**, 782 (1967).

[5] F. Tipler, *Ann. Phys*. **108**, 1 (1977).

[6] G. T. Horowitz, "Topological Change in General Relativity", arXiv:hep-th/9109130. (To appear in the proceedings of the Sixth Marcel Grossman Meeting held in Kyoto, Japan, June 24-29, 1991).

[7] A. Borde, "Topological Change in Classical General Relativity", arXiv:gr-qc/9406053v1 (1994). See also, *Bull. Astr. Soc. India* **25**, 571 (1997).

into a series of spacelike hypersurfaces exist as the topological change occurs. He points out that there exit solutions to the Einstein field equations that do not admit a foliation of spacetime into spacelike hypersurfaces everywhere.

The net result from this literature is that there is no compelling reason to exclude the type of topological change shown in Fig. 2. The issue of the mechanism for the separation of quarks and antiquarks into B_1^3 and B_2^3, just after the radiation dominated period ends at ~4.7×10^4 yr after the big bang, and when the quark-gluon plasma comes into existence, remains to be explained.

Consider the possibility that the quarks and antiquarks in the 3-sphere $\mathbb{S}^3$ are a disjoint union of two distinct subsets, $\{quarks\}$ and $\{antiquarks\}$. A disjoint union is a binary operator, $\sqcup$, that combines these distinct subsets while retaining the distinguishing characteristics of the original sets ($\{quarks\}$ and $\{antiquarks\}$). That is, $\{quarks\} \sqcup \{antiquarks\} = (\{quarks\} \times \{0\}) \cup (\{antiquarks\} \times \{1\})$, where $\times$ is the Cartesian product. In general, the Cartesian product of two sets A and B is defined to be the set $A \times B = \{(a,b) | a \in A \; and \; b \in B\}$. Both $\{0\}$ and $\{1\}$ are sets comprised of one member. The fact that the members here are chosen to be numbers is irrelevant. They could be members of a discrete space for disjoint unions with more than two members.

Now define a mapping function f such that

$$f\colon \{quarks\} \cup \{antiquarks\} \rightarrow \{quarks\} \sqcup \{antiquarks\}.$$

Eq. (3)

This means that if $quark_1 \in \{quarks\}$, then $f(quark_1) = (quark_1, 0)$ and if $quark_2 \in \{antiquarks\}$, then $f(quark_2) = (quark_2, 1)$.

The inverse of the function f is then given by

$$f^{-1}: \{quarks\} \sqcup \{antiquarks\} \rightarrow \{quarks\} \cup \{antiquarks\},$$

Eq. (4)

f^{-1}takes $(quark_1, 0) \longmapsto quark_1 \in \{quarks\}$ while f^{-1}takes $(quark_2, 1) \longmapsto quark_2 \in \{antiquarks\}$.

In general, the inverse image of an open subset $(\mathrm{X} \times \{0\}) \cup (\mathrm{Y} \times \{1\})$ is

$$f^{-1}((\mathrm{X} \times \{0\}) \cup (\mathrm{Y} \times \{1\})) = X \cup Y.$$

Eq. (5)

$f^{-1}$is both a bijection (a one-to-one correspondence) and a homeomorphism. Using the concept of a disjoint union, and defining f^{-1} so as to separate quarks and antiquarks separately into B_1^3 and B_2^3, allows the interpretation of these two 3-balls as separate universes, one containing matter and the other antimatter.

Summary

The FRW metric, including inflation, was used as a model for the early universe. Inflation greatly reduces any spatial curvature. Current astronomical observations and measurements constrain the spatial curvature to be very close to zero, but cannot determine the sign of the curvature. Here, a very slight positive curvature was assumed for the 3-dimensional hypersurface of the metric making it a 3-sphere $\mathbb{S}^3$.

The 3-sphere $\mathbb{S}^3$ can be seen to be the union of two 3-balls by a homeomorphism h, which maps the boundary of B_1^3 onto the boundary of B_2^3 so that $\mathbb{S}^3 = B_1^3 \cup_h B_2^3$. The inverse of this homeomorphism, h^{-1}, was then used to obtain $h^{-1}\mathbb{S}^3 = B_1^3 + B_2^3$.

When time associated with the expansion of the universe is introduced, $\mathbb{S}^3$, B_1^3, and B_2^3 would consequently have time moving in the same direction. At the end of the radiation dominated period topological change was introduced that resulted in quarks being mapped into say B_1^3 and antiquarks into B_2^3. This was achieved by using the concept of a disjoint union of quarks and antiquarks and defining f^{-1} to separate quarks and antiquarks separately into B_1^3 and B_2^3, which would then represent two different universes, one containing matter and the other antimatter. In this way PCT symmetry does not have to be broken to obtain a differing number of baryons and antibaryons.

Appendix1: The Big Bang Model[8]

The standard "big bang" model of cosmology assumes that at the very beginning of the universe there was no matter present but only energy in the form of enormously hot thermal radiation. The actual nature of this radiation, associated with a temperature of similar to 10^{32} °K at the Planck time of 10^{-24} sec, is generally thought to be thermal radiation, which is, of course, of electromagnetic origin. The extremely hot origin of the universe is confirmed by the existence of the isotropic 2.7 °K background radiation. The conversion of this early radiation into particle-antiparticle pairs, as the expanding universe cooled through a series of phase changes, is widely believed to be the source of the matter that exists today. The 2.7 °K background radiation itself comes from a time about half a million years after the initial "singularity", by which time the plasma of ions (primarily hydrogen and helium, as well as electrons and photons) had formed and cooled to the point where it became a transparent gas. There is, however, a fundamental problem with this scenario that has not yet been resolved.

Consider the baryons, ordinary matter which make up about five percent of the mass-energy content of the universe. It is apparent that only a small fraction of this matter survived the annihilation of the particle-antiparticle pairs mentioned above. This means that somehow there must have been a small excess of matter over antimatter before the annihilation occurred. For this to be the case, the symmetry between baryons and antibaryons must be broken. Baryon number conservation must be violated so that the various allowed decay schemes resulting in baryons can lead to a difference between the number of baryons and anti-baryons.

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[8] This section is abstracted from G.E. Marsh, *The Immense Journey*, Appendix A (World Scientific, Singapore 2018).

The criteria for breaking this symmetry was established by Sakharov[9] quite some time ago: both *C* and *CP* invariance must be violated, or otherwise for each process that generates a baryon-antibaryon asymmetry there would be a *C* or *CP* conjugate process that would eliminate the possibility of a net asymmetry; and there must be a departure from thermal equilibrium, or *CPT* invariance, which must hold for any local, relativistic field theory would imply that there would be a balance between processes increasing and decreasing baryon number. There is some confusion in the literature about the meaning of the last requirement with regard to "time". For example, Börner[10] states that, "Loosely speaking, the CPT-invariance of local, relativistic field theories and thermodynamic equilibrium imply the invariance under CP, because in thermodynamic equilibrium there is no arrow of time." Grotz and Klapdor[11] state that only if there is a departure from thermodynamic equilibrium will CP-violating interactions permit ". . . the rates of reactions which lead to the formation of baryons, to be larger than the rates of reactions which lead to antibaryons, but in thermodynamic equilibrium, no time direction is given, and the same would also apply to the inverse reactions."

Both statements argue that in thermodynamic equilibrium there is no Arrow of Time; i.e., no specific time direction. As it stands, this is certainly true, but this Arrow of Time has no relation to the kinematic time reversal transformation. There is often confusion between the Arrow of Time and *T*-violation. As put by Sean Carroll in the November 20, 2012 "blog" of the popular magazine *Discover*, referring to the recent results from BaBar on *T* violation, ". . . the entire phenomenon of *T* violation—**has absolutely nothing to do with that arrow of time** [emphasis in the original]."

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[9] A. D. Sakharov, *Pisma Zh. Eksp. Teor. Fiz*. **5**, 32 (1967) [*JETP Lett*. **5**, 24 (1967)] [*Sov. Phys. Usp*. **34**, 392 (1991)] [*Usp. Fiz. Nauk* **161**, 61 (1991)]. Here *C*, *P*, and *T* are the discrete symmetries associated with charge, parity, and time respectively.

[10] G. Börner, *The Early Universe* (Springer-Verlag, Berlin 1993).

[11] K. Grotz and H.V. Klapdor, *The Weak Interaction in Nuclear, Particle and Astrophysics* (Adam Hilger, Bristol 1990).

With regard to Sakharov's requirement that there be a departure from thermodynamic equilibrium, Kolb and Turner[12] argue that, "The necessary non-equilibrium condition is provided by the expansion of the Universe . . . if the expansion rate is faster than key particle interaction rates, departures from equilibrium can result." Calculations by Kolb and Turner show that only a very small C and CP violation can result in the necessary baryon-antibaryon asymmetry.

Systems in thermodynamic equilibrium while they do not have an Arrow of Time, often called "thermodynamic time", do of course move through time in a direction given by the time asymmetry in the three-dimensional space within which we live.

Because of CPT conservation, it is clear that CP violation means that T-invariance is also violated. These symmetry violations are generally discussed in the context of particle decays. For example, the decay of the K-meson tells us that the violation of T-symmetry is very small. But no matter how small the breaking of time reversal invariance, the fact that it exists at all implies that there is a direction of time in particle physics; i.e., a time asymmetry, which—to reiterate it once again—has nothing to do with the thermodynamic Arrow of Time.

The Feynman Stückelberg interpretation of antiparticles treats them as particles moving backwards in time. While he mostly dealt with electrons and positrons, the interpretation also holds for other particles such as baryons. If the CPT theorem holds, any atom such as hydrogen would have an anti-atom that could be interpreted as that atom moving backwards in

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[12] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, New York 1990).

time. More generally, antimatter can be interpreted as normal matter moving backwards in time. Note that baryons are composed of quarks and antibaryons are composed of antiquarks, and while mesons are composed of quarks and antiquarks, all mesons are unstable with lifetimes from about 10^{-8} s to less than 10^{-22} s; they ultimately decay into stable electrons, neutrinos and photons.

The so-called big bang theory of the origin of the universe has matter and antimatter appearing well after the Planck time of 10^{-44} s. This is when the temperature of the universe was equal to the Planck mass; i.e., $T \sim M_{pl}$ where $M_{pl} = \sqrt{\hbar c/G_N} = 1.22 \times 10^{19}\ Gev$. From the period 10^{-10} to 10^{-6} s, the universe was filled with a quark-gluon plasma[13] where both quarks and antiquarks were present.

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[13] There is a large literature on the nature of the quark-gluon plasma, but none of it is relevant here.

Appendix2: CPT Invariance

The CPT theorem implies that every particle has an antiparticle, that the mass of the particle and antiparticle is equal, and if the particle is unstable its lifetime is the same as its antiparticle. If either *C*, *P*, or *T* is violated there will be a violation of one of the other two. There are a number of different proofs of the *CPT* theorem in the literature. The theorem also holds for quantum field theory.

It has also been shown that if *CPT* invariance is violated in an interacting quantum field theory, then the theory also violates Lorentz invariance.[14] There is an additional complicating factor with regard to gauge invariance. The problem of maintaining gauge invariance in QFT when interactions are present, stem from the now well-known fact that the underlying assumptions of QFT are inconsistent. The essence is contained in Haag's theorem,[15] which is concerned with the interaction picture that forms the basis for perturbation theory. Haag's theorem is important not least for the fact that it identifies the reason regularization and renormalization are needed in QFT; that is, the underlying assumptions of relativistic QFT are inconsistent in the context of interacting systems.

Haag's theorem has been generalized by Hall and Wightman who showed that a single, universal Hilbert space representation exists for describing free or interacting fields.[16] There is also a generalization to free neutral scalar fields of different masses by Reed and Simon

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[14] O. W. Greenberg, *Phys. Rev. Lett.* **89**, 231602.

[15] R. F. Streater and A.S. Wightman, PCT, Spin & Statistics, and all That (W. A. Benjamin, Inc., New York, 1964), Sect. 4–5. See also, P. Roman, *Introduction to Quantum Field Theory* (John Wiley & Sons, Inc., New York, 1968), p. 388.

[16] D. Hall and A.S. Wightman, "A theorem on invariant analytic functions with applications to relativistic quantum field theory", *Matematisk-fysiske Meddelelser* **31**, 1 (1957).

implying that the interaction picture is always inconsistent.[17] In non-relativistic quantum mechanics, there is always a unitary equivalence between the free and interacting representations. For quantum field theory these two representations are unitarily inequivalent. However, as put by Wallace,[18] "algebraic quantum field theory has unitarily inequivalent representations even on spatially finite regions, but this lack of unitary equivalence only manifests itself with respect to expectation values on arbitrary small spacetime regions."

What the above discussion shows is that the idea of breaking the baryon number symmetry between baryons and anti-baryons—resulting in *C* and *CP* invariance being violated in the early universe—so as to obtain a preponderance of matter over antimatter is fraught with problems. For a thorough discussion of matter-antimatter asymmetry in the early universe see Canetti, Drewes, and Shaposhnikov.[19] They conclude, ". . . baryons are the remnant of a small matter-antimatter asymmetry $\sim 10^{-10}$ in the early universe. This asymmetry cannot be explained within the Standard Model of particle physics and cosmology." Another approach is needed.

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[17] M.C. Reed and B. Simon, (*Methods of Modern Mathematical Physics*, *Vol*. II, Academic Press, New York, NY 1975), Theorem X.46.

[18] D. Wallace, "Taking particle physics seriously: A critique of the algebraic approach to quantum field theory",. *Studies in History and Philosophy of Modern Physics* **42 (2)**, pp. 116–125 (2011).

[19] Canetti, L., Drewes, M., and Shaposhnikov, M., "Matter and Antimatter in the Universe", *New J. Phys*. **14**, 095012 (2912), arXiv: 1204.4186.